

Medium Effects in Coherent Pion Photo- and Electroproduction on ${}^4\text{He}$ and ${}^{12}\text{C}$

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Abstract

Coherent π^0 photo- and electroproduction on ${}^4\text{He}$ and ${}^{12}\text{C}$ nuclei is investigated in the framework of a distorted wave impulse approximation in momentum space. The elementary process is described by the recently developed unitary isobar model. Medium effects are considered by introducing a phenomenological Δ self-energy. The recent experimental data for ${}^4\text{He}$ and ${}^{12}\text{C}$ can be well described over a wide range of energies and emission angles by the assumption that the Δ -nuclear interaction saturates.

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I. INTRODUCTION

Recently, much attention has been paid to experimental investigations of coherent π^0 photo- and electroproduction off nuclei, i.e.

$$\gamma^* + A(gs) \rightarrow A(gs) + \pi^0, \quad (1)$$

where γ^* is a real or virtual photon and $A(gs)$ is a nucleus in its ground state. This reaction is of special interest for nuclei with zero spin and isospin. In this case the theoretical treatment simplifies and contains only a minimum number of ingredients. Since the description of the nuclear ground state is well under control, the reaction is then suitable for analyzing medium effects in the production and propagation of Δ resonance.

The earliest theoretical and experimental studies of this reaction had the aim to extract additional information about the elementary amplitude, in particular about the values of the M_{1+} and M_{1-} multipoles, which are related to the excitation of the resonances $\Delta(1232)$ and $N^*(1440)$, respectively. The general interest in this field was revived in the 80's in the context of studying the photoproduction of neutral pions on the proton in order to check the threshold predictions of low energy theorems.

On the other hand, coherent π^0 photoproduction off nuclei was considered as unique tool to examine the properties of the pion-nucleus interaction and to obtain information about the behavior of the pion in the nuclear medium. The theoretical method used for this purpose was the distorted wave impulse approximation (DWIA). While this method was originally formulated in coordinate space with a purely phenomenological optical potential, the modern approach to DWIA uses a representation in momentum space. The first application of the DWIA in the momentum space to pion photoproduction on nuclei was due to Eramzhyan and collaborators [1,2]. Within this approach the pion-nucleus interaction (or final state interaction – FSI) is treated dynamically starting from the elementary pion-nucleon amplitude, and consistently taken into account of nonlocalities of the photoproduction operator and off-shell effects. In particular, this method was applied to the case of coherent π^0 production by Ref. [3]. In the same spirit, the authors of Refs. [4–6] developed their dynamical model for pion photoproduction on the nucleon.

However, FSI is only one aspect of the medium effects due to the modification of the pion propagator in the nuclear medium. Another important issue is the modification of the resonance characteristics (width and position) inside a nucleus. This problem was the subject of numerous investigations, especially in the framework of the Δ -hole model. For a review and further literature, the reader is referred to the monographs [7–9]. This issue will be the main subject of our study. For this purpose we apply, for the first time, the Unitary Isobar Model (UIM) of Ref. [10] to pion photoproduction on nuclei. This recently developed model is particularly adopted to describe the elementary process on the nucleon in the nuclear environment. An important advantage of this model is that the contributions of the background (Born terms and vector meson exchange) and "dressed" Δ resonance are separated in a unitary way. As we will show below this allows us to incorporate medium effects in a self-consistent way with regard to both the "bare" $\gamma N \Delta$ vertices and Δ excitations due to pion rescattering. Another advantage of the UIM is that it covers both pion photoproduction and pion electroproduction up to 4-momentum transfers of $Q^2 = -k^2 \simeq 4 (GeV/c)^2$. Therefore, this model provides quite a unique opportunity to investigate the Q^2 dependence

of medium effects. Note that the first theoretical study of the medium effects in the coherent π^0 electroproduction on the heavy nuclei was done in Ref. [11].

The structure of this paper is as follows. In sections 2 and 3 we consider the main theoretical ingredients to describe pion photo- and electroproduction on nuclei. Our results and predictions are presented in section 4. We shall demonstrate that all the available data for coherent pion photoproduction on ^4He and ^{12}C can be described, in a self-consistent way, with the same parameter set for the Δ self-energy. Finally, our conclusions are summarized in section 5.

II. GENERAL EXPRESSIONS

A. Differential cross sections

In the case of an unpolarized beam and unpolarized target, the 5-fold differential cross section in the *lab* frame can be written as (see Ref. [12] for details)

$$\frac{d\sigma}{d\Omega_{e'} dE_{e'} d\Omega_\pi} = \Gamma \frac{d\sigma_V}{d\Omega_\pi}, \quad (2)$$

which defines the virtual photon cross section

$$\frac{d\sigma_V}{d\Omega_\pi} = \frac{d\sigma_T}{d\Omega_\pi} + \epsilon \frac{d\sigma_L}{d\Omega_\pi} + \sqrt{2\epsilon(1+\epsilon)} \frac{d\sigma_{TL}}{d\Omega_\pi} \cos \Phi_\pi + \epsilon \frac{d\sigma_{TT}}{d\Omega_\pi} \cos 2\Phi_\pi, \quad (3)$$

with ϵ and Γ the degree of transverse polarization and the flux of the virtual photon, respectively. Denoting the photon's *lab* energy by $\omega_L = E_e - E_{e'}$ and its three-momentum by \mathbf{k}_L , we have

$$\epsilon = \left[1 + 2 \frac{\mathbf{k}_L^2}{Q^2} \tan^2 \frac{\theta_e}{2} \right]^{-1}, \quad \Gamma = \frac{\alpha}{2\pi^2} \frac{E_{e'}}{E_e} \frac{K}{Q^2} \frac{1}{1-\epsilon}. \quad (4)$$

In accordance with our previous work, we define the flux by the photon "equivalent energy" $K = (s_A - M_A^2)/2M_A$, with the Mandelstam variable $s_A = E^2 = -Q^2 + M_A^2 + 2\omega_L M_A$, M_A the mass of the nuclear target, and the square of the four-momentum transfer $Q^2 = -k^2 = \mathbf{k}_L^2 - \omega_L^2 > 0$.

The first two terms in Eq. (3) are the transverse (T) and longitudinal (L) cross sections, and the third and fourth terms are the transverse-longitudinal (TL) and transverse-transverse (TT) interference cross sections. The latter ones can be separated by use of their typical dependence on the azimuthal angle ϕ_π of pion emission. As in the case of pion electroproduction on the free nucleon, these four cross sections may be expressed in terms of the nuclear tensor $W_{\mu\nu}$, with μ and ν corresponding to the Cartesian coordinates x, y or z , which leads to the following relations in the *cm* frame:

$$\frac{d\sigma_T}{d\Omega_\pi} = \frac{k_\pi}{2k_\gamma^{cm}} (W_{xx} + W_{yy}), \quad \frac{d\sigma_{TT}}{d\Omega_\pi} = \frac{k_\pi}{2k_\gamma^{cm}} (W_{xx} - W_{yy}), \quad (5a)$$

$$\frac{d\sigma_L}{d\Omega_\pi} = \frac{k_\pi}{k_\gamma^{cm}} \frac{Q^2}{\omega_\gamma^2} W_{zz}, \quad \frac{d\sigma_{TL}}{d\Omega_\pi} = -\frac{k_\pi}{k_\gamma^{cm}} \frac{Q}{\omega_\gamma} \text{Re} W_{xz}, \quad (5b)$$

with $k_\gamma^{cm} = (E^2 - M_A^2)/2E$ the "photon equivalent energy" in the cm frame and $k_\pi = |\mathbf{k}_\pi|$ the pion momentum. Note that we use a right-handed coordinate system with the positive z -axis along the virtual photon momentum \mathbf{k}_γ and the y -axis along the vector $[\mathbf{k}_\gamma \times \mathbf{k}_\pi]$. The nuclear tensor $W_{\mu\nu}$ is defined in terms of the nuclear transition amplitudes $F_{fi}^{(\mu)}$ describing the transitions from the initial states $|i\rangle = |J_i M_i\rangle$ to the final states $|f\rangle = |J_f M_f\rangle$ with spin $J_{i(f)}$ and projection $M_{i(f)}$. After summing and averaging over the nuclear spin degrees of freedom we obtain

$$W_{\mu\nu} = \frac{1}{2J_i + 1} \text{Re} \sum_{M_i, M_f} F_{fi}^{(\mu)} F_{fi}^{(\nu)*}. \quad (6)$$

For the purpose of the numerical calculations it is convenient to express the nuclear amplitudes in the covariant spherical basis $\mathbf{e} = \{\mathbf{e}_{+1}, \mathbf{e}_{-1}, \mathbf{e}_0\}$ by the relations

$$F_{fi}^{(\lambda)} = -\frac{\lambda}{\sqrt{2}}(F_{fi}^{(x)} + i\lambda F_{fi}^{(y)}), \quad F_{fi}^{(0)} = F_{fi}^{(z)}, \quad (7)$$

where $\lambda = \pm 1$ is the helicity of the transverse photon.

In the case of targets with zero spin, the general symmetry properties and the pseudoscalar nature of the pion have the consequence that the transverse nuclear amplitude $F_{fi}^{(\lambda)}$ contains only a term proportional to $[\mathbf{k}_\pi \times \mathbf{k}_\gamma]$ in the pion-nucleus cm frame. Hence

$$F_{fi}^{(\lambda)}(\mathbf{k}_\pi, \mathbf{k}_\gamma; Q^2) = F_0(\mathbf{k}_\pi, \mathbf{k}_\gamma; Q^2) [\hat{\mathbf{k}}_\pi \times \hat{\mathbf{k}}_\gamma] \cdot \mathbf{e}_\lambda, \quad F_{fi}^{(0)} = 0, \quad (8)$$

where $\hat{\mathbf{k}}_\pi$ and $\hat{\mathbf{k}}_\gamma$ are unit vectors in the directions of the pion and the virtual photon momentum, respectively.

In the right-handed coordinate system with the z -axis along $\hat{\mathbf{k}}_\gamma$ and the y -axis along $[\hat{\mathbf{k}}_\gamma \times \hat{\mathbf{k}}_\pi]$, only the W_{yy} tensor component remains. Therefore, $d\sigma_T/d\Omega_\pi = -d\sigma_{TT}/d\Omega_\pi$ and all other cross sections in Eq. (3) vanish. In this case the expression for the virtual photon cross section simplifies to [11]

$$\frac{d\sigma_V}{d\Omega_\pi} = \frac{d\sigma_T}{d\Omega_\pi} (1 - \epsilon \cos 2\Phi_\pi). \quad (9)$$

It should be noted that Eqs. (8-9) are general. As a consequence the differential cross section has a $\sin^2 \theta_\pi$ and $(1 - \epsilon \cos 2\Phi_\pi)$ dependence in all cases, independently of the reaction mechanism. In fact, this peculiarity of coherent pion photo- and electroproduction on spin zero nuclei is often used to separate coherent and incoherent contributions, or to measure the degree of photon polarization, because the photon asymmetry $\Sigma = -d\sigma_{TT}/d\sigma_T$ equals unity.

B. PWIA and DWIA nuclear amplitudes

Let us first consider the nuclear amplitude in a simple plane wave impulse approximation (PWIA). The corresponding amplitude is denoted by $V_{fi}^{(\lambda)}$. It can be expressed in terms of the elementary pion electroproduction amplitudes $f_{\gamma\pi}^{(\lambda)}$ and reduced to the form

$$V_{fi}^{(\lambda)}(\mathbf{k}_\pi, \mathbf{k}_\gamma; Q^2) = \mathcal{W}_A \int d\mathbf{r} \Psi_f^*(\mathbf{r}) \sum_{j=1}^A e^{i\mathbf{q}\cdot\mathbf{r}_j} f_{\gamma\pi}^{(\lambda)}(\mathbf{k}_\pi, \mathbf{k}_\gamma, \mathbf{p}_j; Q^2) \Psi_i(\mathbf{r}) \quad (10)$$

with $\mathbf{r} = \{\mathbf{r}_1, \mathbf{r}_2, \dots, \mathbf{r}_A\}$ the set of nucleon coordinates, a phase space factor \mathcal{W}_A given by

$$\mathcal{W}_A = \frac{W}{E} \sqrt{\frac{E_A(k_\pi)E_A(k_\gamma)}{E_N(p')E_N(p)}}, \quad (11)$$

\mathbf{p} and \mathbf{p}' the initial and final nucleon momenta, $E_{A(N)}$ the nuclear (nucleon) energies, $\mathbf{q} = \mathbf{k}_\gamma - \mathbf{k}_\pi$ the momentum transfer to the nucleus, and $Q^2 = \mathbf{k}_\gamma^2 - \omega_\gamma^2$ expressed by the cm momentum and energy of the virtual photon.

The invariant amplitude $f_{\gamma\pi}^{(\lambda)}$ describes the elementary process on the free nucleon. It is evaluated within the framework of the UIM in terms of the standard CGLN amplitudes $F_i(W, \cos \tilde{\theta}_\pi, Q^2)$, with the pion angle $\tilde{\theta}_\pi$ and the total energy W in the γN cm system,

$$W = \sqrt{(\omega_\gamma + E_N(\mathbf{p}))^2 - (\mathbf{k}_\gamma + \mathbf{p})^2}. \quad (12)$$

Note that in the literature there are several prescriptions for the relations between total pion-nuclear energy E and W with various physical or mathematical motivations (see for example Refs. [13–17]). Using this freedom we in principle can effectively take into account medium effects and improve by this way (or "optimize") impulse approximation. However, our main aim is the study the medium effects explicitly. Therefore, in the present work we will use physically transparent relation (12) which does not contain information about residual nucleus and about final state.

The next problem is related to the treatment of Fermi motion, i.e. the dependence of the amplitudes on the nucleon's momentum \mathbf{p} . Previous studies of pion-nucleus scattering [13,14] and pion photoproduction reactions [18,19] showed that a good approximation to the proper averaging over Fermi motion is given by the "factorization approximation", which supposes that the main part of Fermi motion can be accounted for by evaluating the elementary amplitude at "effective" nucleon momenta \mathbf{p} and \mathbf{p}' in the initial and final states,

$$\mathbf{p} = -\frac{\mathbf{k}_\gamma}{A} - \frac{A-1}{2A}\mathbf{q}, \quad \mathbf{p}' = -\frac{\mathbf{k}_\pi}{A} + \frac{A-1}{2A}\mathbf{q}, \quad (13)$$

where A is the number of nucleons. The same approximation, but in the nuclear rest frame is taken in Refs. [20,21]. Note that this approximation is based on the fact that for Gaussian nuclear wave functions (which reproduce the ground state of light nuclei sufficiently well) the replacement leads to an exact treatment of the terms linear in $\mathbf{p}/2M$ in the elementary amplitude. Moreover, it allows a consistent description of nuclear and nucleon kinematics, by means of a simultaneous conservation of energies and momenta for both the pion-nucleus and the pion-nucleon systems. As an example this factorization approximation was tested numerically for pion photoproduction on p-shell nuclei [19]. Recently this approximation was also examined within a more elaborate relativistic model for coherent pion photoproduction on ^{12}C [22].

In the case of coherent photo- and electroproduction of neutral pions on spin and isospin zero nuclei, the factorization approximation allows us to obtain the following simple expression for the PWIA amplitude $V_{fi}^{(\lambda)}$

$$V_{fi}^{(\lambda)}(\mathbf{k}_\pi, \mathbf{k}_\gamma; Q^2) = A \mathcal{W}_A f_2(\mathbf{k}_\pi, \mathbf{k}_\gamma; Q^2) F_A(q) [\hat{\mathbf{k}}_\pi \times \hat{\mathbf{k}}_\gamma] \cdot \mathbf{e}_\lambda, \quad (14)$$

where $f_2 = [f_2(p\pi^0) + f_2(n\pi^0)]/2$ is the isoscalar non spin-flip part of the elementary amplitude boosted to the pion-nucleus cm frame. The nuclear form factor is normalized to $F_A(0) = 1$. It can be extracted from the nuclear charge form factor by the relation $F_A^{ch}(q) = F_A(q)F_p^{ch}(q)$, where $F_p^{ch}(q)$ is the proton charge form factor. The nuclear form factor $F_A(q)$ used in our present work is taken from Ref. [3].

Let us now consider the full DWIA amplitude including pion distortion or FSI effects. In momentum space it takes the form [1]

$$F_{fi}^{(\lambda)}(\mathbf{k}_\pi, \mathbf{k}_\gamma; Q^2) = V_{fi}^{(\lambda)}(\mathbf{k}_\pi, \mathbf{k}_\gamma; Q^2) + D_{fi}^{(\lambda)}(\mathbf{k}_\pi, \mathbf{k}_\gamma; Q^2), \quad (15)$$

where $D_{fi}^{(\lambda)}$ is the contribution from the pion-nucleus interaction expressed in terms of the pion-nucleus elastic scattering amplitude $F_{\pi A}$,

$$D_{fi}^{(\lambda)}(\mathbf{k}_\pi, \mathbf{k}_\gamma; Q^2) = -\frac{a}{(2\pi)^2} \sum_{M'_f} \int \frac{d\mathbf{k}'_\pi}{\mathcal{M}(k'_\pi)} \frac{F_{\pi A}(\mathbf{k}_\pi, \mathbf{k}'_\pi) V_{fi}^{(\lambda)}(\mathbf{k}'_\pi, \mathbf{k}_\gamma; Q^2)}{E(k_\pi) - E(k'_\pi) + i\epsilon}. \quad (16)$$

In this equation, the relativistic reduced mass of the pion-nucleus system is given by $\mathcal{M}(k_\pi) = \omega_\pi(k_\pi)E_A(k_\pi)/E(k_\pi)$. Note that the main difference compared to standard DWIA is the factor $a = (A-1)/A$, which eliminates double counting of pion rescattering on one and the same nucleon. Such effects are in fact already included in the elementary amplitude. Finally, the pion scattering amplitude $F_{\pi A}$ is constructed as solution of a Lippmann-Schwinger integral equation (for details see Refs. [23,24]).

III. ELEMENTARY AMPLITUDE AND Δ SELF-ENERGY

A. Elementary amplitudes on- and off-shell

The detailed information about the elementary reaction amplitude for the free nucleon is given in Ref. [10]. In the coherent π^0 photo- and electroproduction on spin zero nuclei only the spin independent part contributes. Therefore, in the pion-nucleon cm frame we have

$$\tilde{f}_{\gamma\pi}^{(\lambda)} = F_2(\tilde{\mathbf{k}}_\pi, \tilde{\mathbf{k}}_\gamma; W, Q^2) [\hat{\mathbf{k}}_\pi \times \hat{\mathbf{k}}_\gamma] \cdot \mathbf{e}_\lambda, \quad (17)$$

where F_2 is the standard CGLN amplitude and $\tilde{\mathbf{k}}_{\gamma(\pi)}$ is the photon (pion) momentum in the πN cm frame. These momenta can be boosted to an arbitrary frame by the Lorentz transformation

$$\tilde{\mathbf{k}}_{\gamma(\pi)} = \mathbf{k}_{\gamma(\pi)} + \alpha_{\gamma(\pi)} \mathbf{P}, \quad \alpha_{\gamma(\pi)} = \frac{1}{W_{i(f)}} \left(\frac{\mathbf{P} \cdot \mathbf{k}_{\gamma(\pi)}}{E_{i(f)} + W_{i(f)}} - \omega_{\gamma(\pi)} \right), \quad (18)$$

where $E_{i(f)} = \omega_{\gamma(\pi)} + E_N^{i(f)}$ and $W_{i(f)}$ are the total energies for the photon-nucleon (pion-nucleon) systems in the arbitrary and cm frames, respectively. The total momentum $\mathbf{P} = \mathbf{k}_\gamma + \mathbf{p} = \mathbf{k}_\pi + \mathbf{p}'$ is given in the arbitrary frame. Using the factorization approximation (13)

and the transformation (18), we can get a simple connection between the f_2 amplitude of Eq. (14) with the CGLN amplitude F_2 ,

$$f_2 = \frac{k_\gamma k_\pi}{\tilde{k}_\gamma \tilde{k}_\pi} \left[1 + \frac{A-1}{2A} (\alpha_\gamma + \alpha_\pi) \right] F_2. \quad (19)$$

Within the UIM the amplitude F_2 contains contributions from Born and vector meson exchange terms, $F_2^{(B)}$, and from "dressed" N^* resonances, $F_2^{(R)}$. In our present work we consider the photon's *lab* energy range $E_\gamma < 400$ MeV, for which the s-channel $\Delta(1232)$ resonance gives the most important contribution. The multipole $M_{1+}^{(3/2)}$ related to the resonance is parametrized in a unitary way by a standard Breit-Wigner form, i.e. $F_2^{(R)} = 2M_{1+}^{(\Delta)} + M_{1-}^{(Roper)}$, where

$$M_{1+}^{(\Delta)}(W, Q^2) = F_{\gamma N \Delta}(W, Q^2) \frac{\Gamma_\Delta(W) M_\Delta e^{i\phi}}{M_\Delta^2 - W^2 - iM_\Delta \Gamma_\Delta} F_{\pi N \Delta}(W), \quad (20)$$

and $M_{1-}^{(Roper)}$ accounts for the small contribution of the Roper resonance $N^*(1440)$. In Eq. (20), $F_{\gamma N \Delta}$ and $F_{\pi N \Delta}$ are the $\gamma N \Delta$ and $\pi N \Delta$ vertex function, respectively, M_Δ and $\Gamma_\Delta(W)$ the energy and total width of the Δ resonance as defined in Ref. [10]. The unitary phase $\phi(W, Q^2)$ adjusts the phase of the total multipole (background+resonance) to the corresponding pion-nucleon scattering phase, in accordance with the Fermi-Watson theorem.

As has been mentioned in the Introduction, it is one of the advantages of the DWIA approach in momentum space that it allows us to account for the nonlocality of the elementary operator and off-shell effects. Such effects originate mainly from the principal value part of the Eq. (16), the nonlocality being related with the dependence of the amplitude on the pion momentum \mathbf{k}_π . For the background contribution $F_2^{(B)}$, this dependence is in principle fixed by the form of the Lagrangians. However, the UIM background contribution is derived from Lagrangians with a zero-range interaction. In this case the standard way (often used in coupled channels [25], dynamical [6] or meson exchange [4] models) is to introduce the finite range aspects of the interaction by an off-shell form factor. In this spirit we have multiplied the background contribution by a dipole-like form factor

$$f(\tilde{k}_\pi, \tilde{k}_W) = \left(\frac{\Lambda^2 + \tilde{k}_W^2}{\Lambda^2 + \tilde{k}_\pi^2} \right)^2, \quad (21)$$

where \tilde{k}_W is the on-shell value of the pion momentum in the πN *cm* frame corresponding to the total energy W for the γN system. The standard value for the cut-off parameter Λ , which provides the best fit in many calculations of pion-nucleon scattering and pion photoproduction, is in the range of 400-500 MeV. In our present work we shall fix this parameter at $\Lambda = 450$ MeV. For consistency with our off-shell extrapolation of the pion-nucleon scattering amplitude in the P_{33} channel [23], we shall use the prescription

$$M_{1+}^{(\Delta)}(\tilde{k}_\pi, W, Q^2) = M_{1+}^{(\Delta)}(W, Q^2) \frac{\tilde{k}_\pi}{\tilde{k}_W} f(\tilde{k}_\pi, \tilde{k}_W) \quad (22)$$

for the Δ contribution, with the factor $\tilde{k}_\pi/\tilde{k}_W$ providing the correct threshold behavior in the off-shell case.

B. Self-energy of the Δ

It has been known from numerous Δ -hole model calculations of pion scattering, pion photoproduction and photoabsorption reactions that the properties of the bound Δ isobar in the nuclear medium differ substantially from those of the free Δ (see Ref. [8] and references therein).

In the case of pion photo- and electroproduction, two main mechanisms excite the Δ isobar excitation, i) a direct excitation with a "bare" $\gamma N\Delta$ vertex, which corresponds to diagram (b) in Fig. 1, and ii) a vertex renormalization mechanism, for which the Δ is excited with pions produced by background terms (diagram (c) in Fig. 1). As has been shown recently [26] contributions of these two diagrams are equally important and, therefore, medium effects have to account for both mechanisms. The natural way to do so is to introduce medium effects in the form of dressed resonance contributions (diagram (a)) as defined by the UIM and given by Eq. (20). However, as a first step, the relativistic Δ propagator with the unitary phase ϕ has to be reduced to the nonrelativistic Breit-Wigner form. This can be achieved by the transformation

$$\frac{e^{i\phi}}{M_\Delta^2 - W^2 - iM_\Delta\Gamma_\Delta(W)} = -\frac{1}{W + M_\Delta} \cdot \frac{1}{W - \bar{M}_\Delta(W) + i\bar{\Gamma}_\Delta(W)/2}, \quad (23)$$

where

$$\bar{M}_\Delta(W) = W - (W - M_\Delta) \cos \phi - \frac{M_\Delta\Gamma_\Delta(W)}{W + M_\Delta} \sin \phi, \quad (24a)$$

$$\bar{\Gamma}_\Delta = \frac{2M_\Delta\Gamma_\Delta(W)}{W + M_\Delta} \cos \phi - 2(W - M_\Delta) \sin \phi. \quad (24b)$$

The next step is to modify DWIA amplitude by introducing the so-called Δ self-energy Σ_Δ on the *rhs* of Eq. (23), i.e.

$$\frac{1}{W - \bar{M}_\Delta + i\bar{\Gamma}_\Delta(W)/2} \rightarrow \frac{1}{W - \bar{M}_\Delta + i\bar{\Gamma}_\Delta(W)/2 - \Sigma_\Delta}. \quad (25)$$

In the literature we find many approaches to calculate Σ_Δ within simple models based on the local density approximation [27,28] or more refined microscopical calculations [9,29] based on the Δ -hole model. Such models were quite successful in describing pion scattering and pion photoproduction on heavy nuclei. However, one of the peculiarities of these approaches is that they finally always incorporate elements of phenomenology. In our present work we will do so from the very beginning by looking for a phenomenological parametrization of Σ_Δ which should be simple, common for all nuclei and able to describe the available data. For this purpose we shall test two types of parametrization

$$\Sigma_\Delta(E_\gamma, q^2) = V_1(E_\gamma) F(q^2), \quad F(q^2) = e^{-\beta q^2}, \quad (26a)$$

and

$$\Sigma_\Delta(E_\gamma, r) = (A - 1) V_2(E_\gamma) \rho(r) / \rho_0, \quad \rho_0 = 0.17 fm^{-3}, \quad (26b)$$

where $V_{1,2}$ is a (complex and energy-dependent) free parameter. The first parametrization, Eq. (26a), is convenient to use in momentum space. Here we assume that $F(q^2)$ is the s -shell harmonic oscillator form factor with $\beta = 0.54 \text{ fm}^2$ and that Σ_Δ is already saturated for ${}^4\text{He}$. This has the consequence that V_1 and $F(q^2)$ have the same values for both ${}^4\text{He}$ and ${}^{12}\text{C}$. The second parametrization, Eq. (26b), results from the local density approximation, the nuclear density $\rho(r)$ being normalized to $\int \rho(r) d^3r = 1$. Therefore, the Δ self-energy will differ for ${}^4\text{He}$ and ${}^{12}\text{C}$.

Finally we note that in the present context we neglect medium effects due to the Roper resonance, because this contribution is very small in our energy range, i.e. of the order of 2-3% at a photon's *lab* energy $E_\gamma < 400 \text{ MeV}$.

IV. RESULTS AND DISCUSSION

One of the attractive features of coherent pion photo- and electroproduction on nuclei is that we can obtain clear signals from modifications of the Δ isobar in the nuclear medium. For this purpose, Fig. 2 presents our results for the reaction ${}^4\text{He}(\gamma, \pi^0){}^4\text{He}$ and ${}^{12}\text{C}(\gamma, \pi^0){}^{12}\text{C}$ at a photon's *lab* energy $E_\gamma = 290 \text{ MeV}$. In this energy region the standard DWIA calculations substantially overestimate the measured differential cross sections. Our DWIA results (dashed curves) in the maximum are larger than the experimental data by approximately a factor of 2. As was shown in previous studies, one of the reasons for this large discrepancy is the interaction of the Δ isobar with the surrounding nucleons.

The Δ -nucleus interaction leads to a renormalization of the Δ propagator and can be described in terms of the Δ self-energy Σ_Δ . The recently measured differential cross sections for the reaction ${}^4\text{He}(\gamma, \pi^0){}^4\text{He}$ and forthcoming new data for ${}^{12}\text{C}(\gamma, \pi^0){}^{12}\text{C}$ to be measured over a wide energy region ($200 < E_\gamma < 400 \text{ MeV}$) provide us with a unique opportunity to determine Σ_Δ with good accuracy. As has been pointed out in the last section, we shall now test two types of parametrizations given by Eq. (26).

First, let us consider ρ -type parametrization (26b) conventionally used in the analysis of elastic pion-nucleus scattering. Note that a Taylor expansion of Eq. (25) in $\rho(r)$ leads to medium corrections in $\rho^2(r)$, which were often used in analyses of pion-nucleus scattering. One of the consequences of this parametrization is that for finite nuclei it gives a good description of the differential cross sections in forward direction but fail at large angles and in the resonance region. On the other hand pion scattering on light nuclei at backward angles is usually better described without the phenomenological ρ^2 term (or without Δ medium effects).

From Fig. 2 we can see that the coherent π^0 photoproduction situation is similar to the case described above (see the dash-dotted curves). This result clearly indicates that for light nuclei the treatment of the Δ -nucleus interaction in terms of $\rho(r)$ becomes less satisfactory. Therefore, nuclei like ${}^4\text{He}$ require a more sophisticated microscopical calculation of the Δ self-energy, e.g. the Δ -hole [9,29] or meson exchange models [35]. We would like to stress that the first microscopical Δ -hole calculation performed some 15 years ago by J. Koch and E. Moniz [29] gives excellent agreement with the results of the recent measurements (see the dotted curve in Fig. 2).

However, a consistent extension of the microscopical calculations to heavier nuclei meets serious difficulties, and at some level it always requires elements of phenomenology. There-

fore, we decide to use another approach in our present work. First, with the use of the F -type parametrization (26a) in momentum space, we extracted Σ_Δ from the data for the reaction ${}^4\text{He}(\gamma, \pi^0){}^4\text{He}$. Then, we assume that the Δ -nucleus interaction is the same also for the heavier nuclei, i.e., that it saturates already for ${}^4\text{He}$. Our calculations at $E_\gamma = 290$ MeV indicate that this is indeed a realistic assumption, and allows us to describe the coherent π^0 photoproduction on ${}^4\text{He}$ and ${}^{12}\text{C}$ over a wide range of pion angles (see the solid curves in Fig. 2). The obtained values for the strength of the Δ -nucleus interaction, $\text{Re } V_1=19$ MeV and $\text{Im } V_1= -33$ MeV, are in reasonable agreement with the values obtained in pion-nucleus scattering.

This result inspires us to test our model at other energies assuming that V_1 is an energy dependent function to be determined by fitting the experimental data for ${}^4\text{He}$. In Fig. 3 we present results of our fit in the energy range $200\text{ MeV} < E_\gamma < 400$ MeV. In Fig. 4 the prediction of our model is compared with the new experimental data from Ref. [33]. The energy dependence for the total cross section and the parameters V_1 are shown in Fig. 5. These results for the potential parameter V_1 agree qualitatively well with the predictions of Ref. [36].

Our most interesting result is shown in Fig. 6, where the prediction is compared to the data of the A2 collaboration at MAMI for the reaction ${}^{12}\text{C}(\gamma, \pi^0){}^{12}\text{C}$ obtained at a pion angle $\theta_\pi = 60^\circ$. Even at such a large angle where the differential cross section has dropped by more than an order of magnitude (see Fig. 2), our assumption about the saturation of the Δ -nucleus interaction is still quite acceptable. Of course, a better understanding of this phenomenon will require additional data at smaller angles where the differential cross section reaches a maximum, as well as data for other nuclei.

A new feature of the Δ -nucleus interaction can be investigated and new information about the Q^2 dependence of Σ_Δ can be obtained by using virtual photons (or coherent π^0 electroproduction). Clearly, as the Δ isobar should not remember how it was excited, the V_1 does not depend on Q^2 . Therefore, the strength of its interaction with the surrounding nucleons is universal for all processes (electroproduction, pion scattering, Compton scattering, etc.) and the Q^2 dependence enters the parametrization of Eq. (26a) only in the form factor $F(q)$, with $q = |\mathbf{k}_\gamma - \mathbf{k}_\pi|$ the momentum transferred to the nucleus and \mathbf{k}_γ the virtual photon momentum as function of Q^2 ,

$$\mathbf{k}_\gamma^2 = Q^2 + \frac{(s_A - M_A^2 - Q^2)^2}{4s_A} = Q^2 + \omega_\gamma^2. \quad (27)$$

In Fig. 7 we depict the predictions of our model for: i) an equivalent photon energy $E_\gamma=228$ MeV and $Q^2=0, 0.054, 1.81$ $(\text{GeV}/c)^2$, and ii) $E_\gamma=289$ MeV and $Q^2=0, 0.074, 1.61$ $(\text{GeV}/c)^2$. This corresponds to the kinematics of the recent and forthcoming data from NIKHEF [38]

V. CONCLUSION

In this paper we have presented a first nuclear application of our recently developed unitary isobar model to pion photo- and electroproduction. This model describes all the existing data for (γ, π) and $(e, e'\pi)$ on the nucleon reasonably well and furthermore compares very well with the partial wave analysis of the VPI group up to $W_{cm} = 1700\text{ MeV}$.

We have performed a DWIA calculation for coherent π^0 photo- and electroproduction from nuclei and applied it to experimentally investigated reactions on ${}^4\text{He}$ and ${}^{12}\text{C}$. Due to the very precise data recently measured at Mainz over a large angular range of photon *lab* energies from 207 MeV up to 397 MeV, we are able to determine, for the first time, the energy dependence of the Δ -nucleus potential from pion photoproduction reactions. Comparing ${}^4\text{He}$ and ${}^{12}\text{C}$ we find no A-dependence of the potential and conclude that the Δ -nucleus interaction saturates already for ${}^4\text{He}$.

In our study of the dynamical mechanism of the Δ self-energy in the nuclear medium we find that a form factor type Δ -nucleus interaction is preferred as compared to a density type of medium modification often used in the local density approximation of coordinate space calculations.

First experimental results on electroproduction obtained at NIKHEF are in good agreement with our calculations and no readjustment of the Δ -nucleus interaction is needed.

For future experiments it will be very interesting to see how the Δ -nucleus interaction saturates by measurements on the deuteron and on ${}^3\text{He}$ and, secondly, if the saturation observed for ${}^{12}\text{C}$ is valid for the heavier nuclei.

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FIGURES

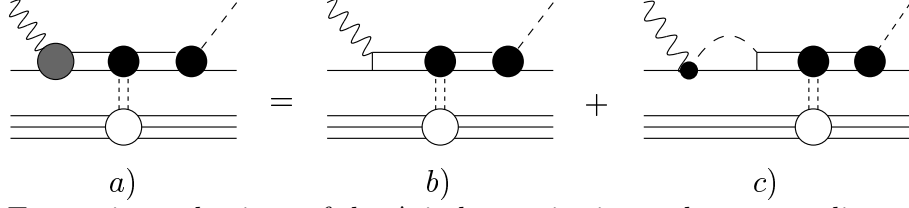


FIG. 1. Two main mechanisms of the Δ isobar excitation and corresponding medium effects: b) direct excitation with "bare" $\gamma N \Delta$ vertex; c) vertex renormalization mechanism where Δ isobar is excited from pions produced by nonresonant background.

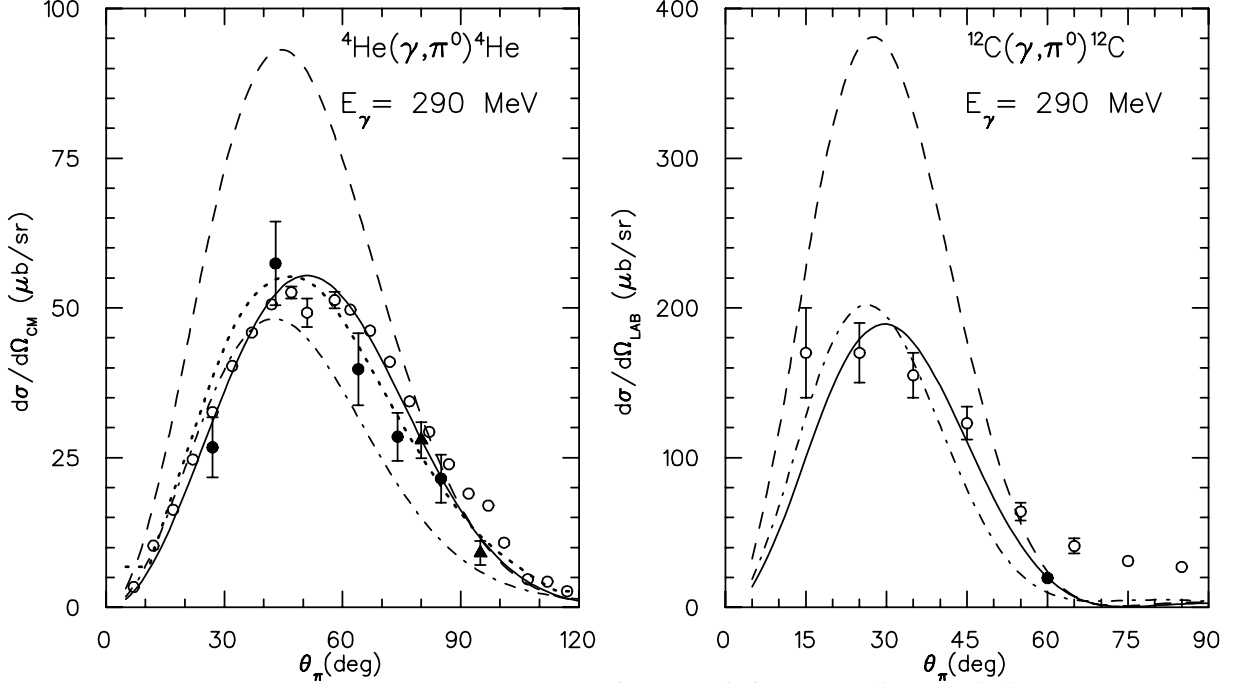


FIG. 2. The differential cross sections for the ${}^4\text{He}(\gamma, \pi^0){}^4\text{He}$ and ${}^{12}\text{C}(\gamma, \pi^0){}^{12}\text{C}$ reactions at photon *lab* energy $E_\gamma = 290$ MeV. Dashed curves are the DWIA results. Dash-dotted and solid curves are the results obtained with ρ -type (26b) and F -type (26a) parametrizations for the Δ self-energy. Dotted curve is the Δ -hole model calculation taken from Ref. [29] and recalculated in the $\pi^4\text{He}$ c.m. frame. Experimental data for ${}^4\text{He}$ are from Refs. [30] (open circles), [31] (full circles) and [32] (full triangles). Experimental data for ${}^{12}\text{C}$ are from Refs. [34] (open circles) and [37] (full circles).

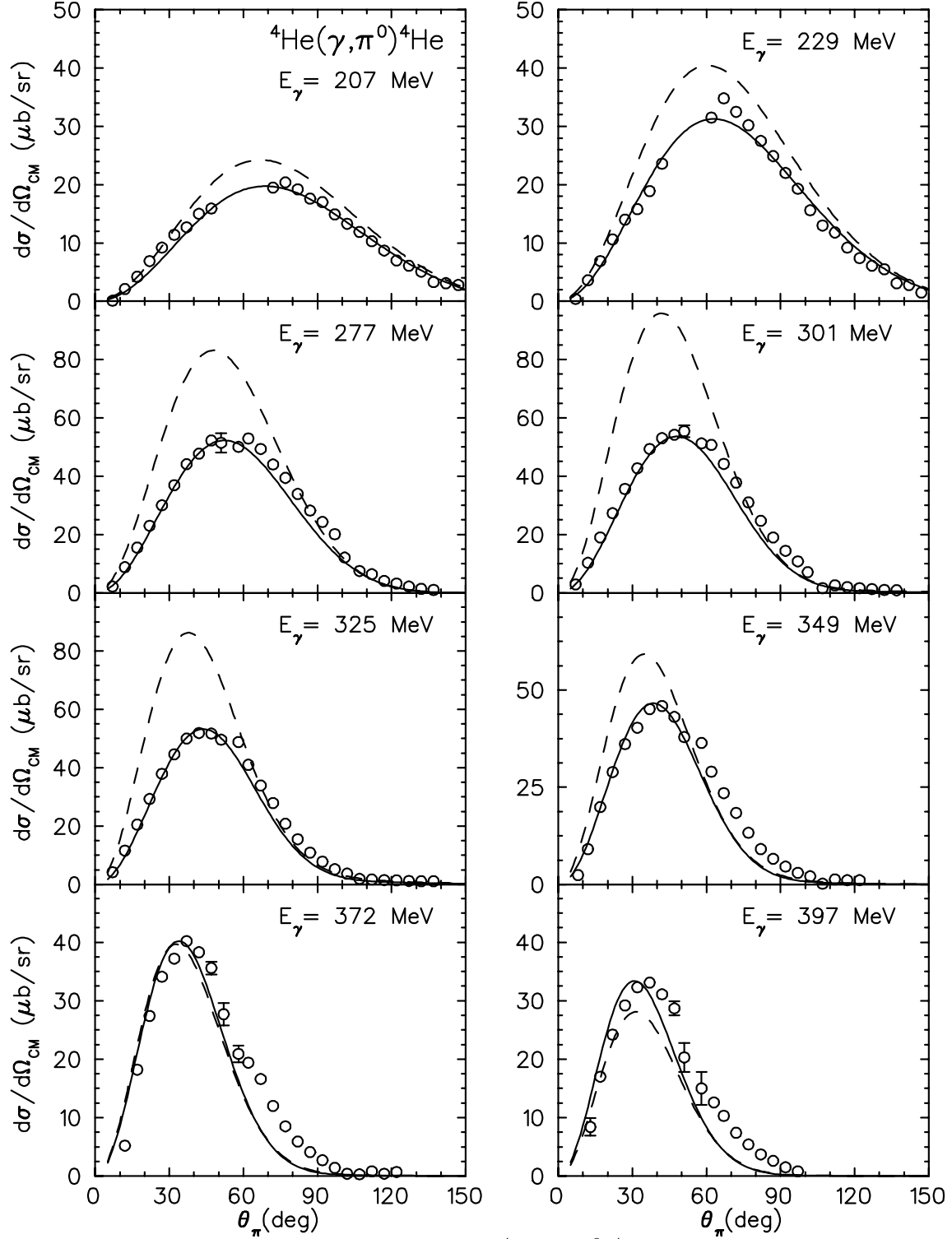


FIG. 3. The differential cross sections for the ${}^4\text{He}(\gamma, \pi^0){}^4\text{He}$ reaction. Dashed curves are the DWIA results. Solid curves are the results obtained with F - type (26a) parametrizations for the Δ self-energy. Experimental data are from Ref. [30].

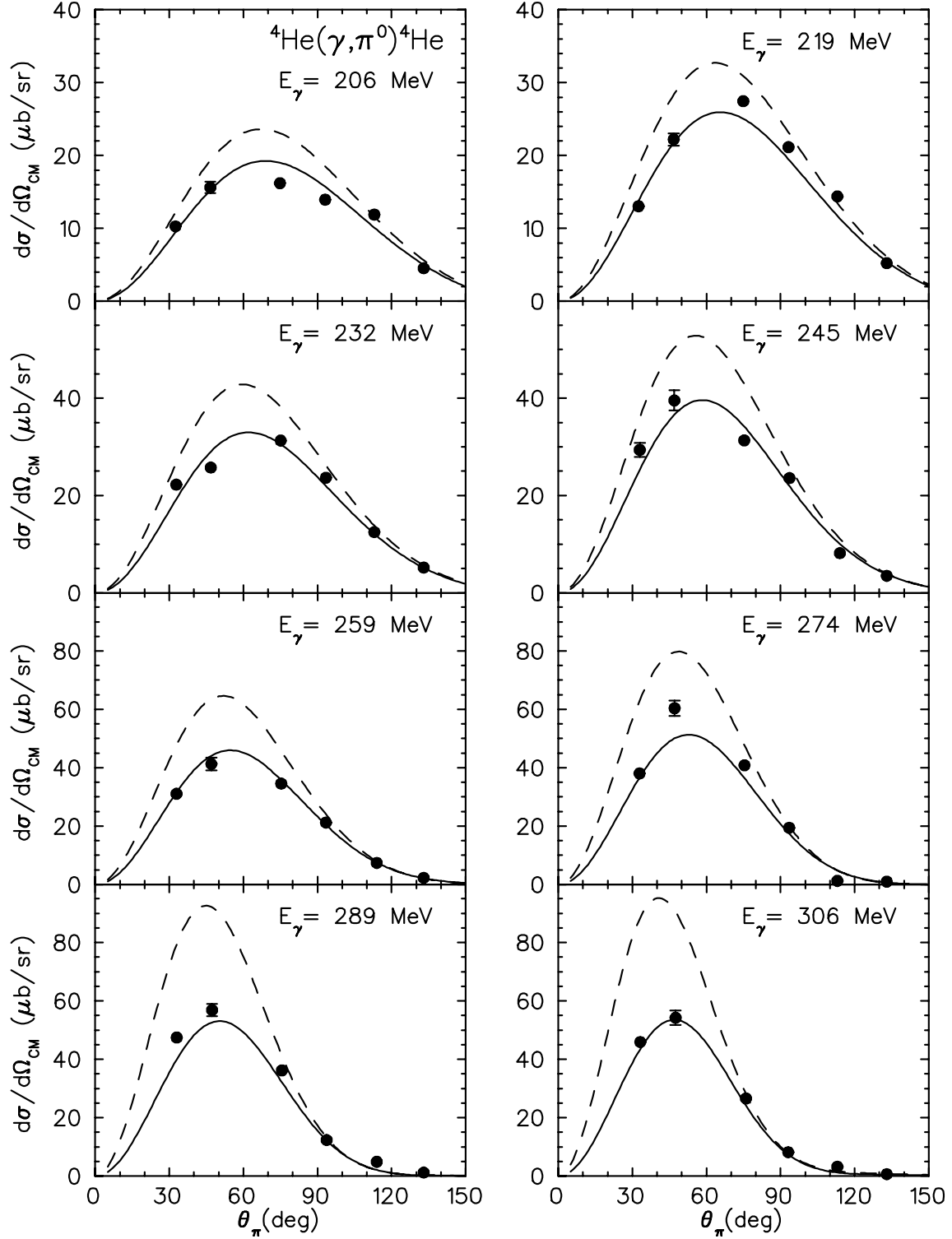


FIG. 4. The same as in Fig. 3. Experimental data are from Ref. [33].

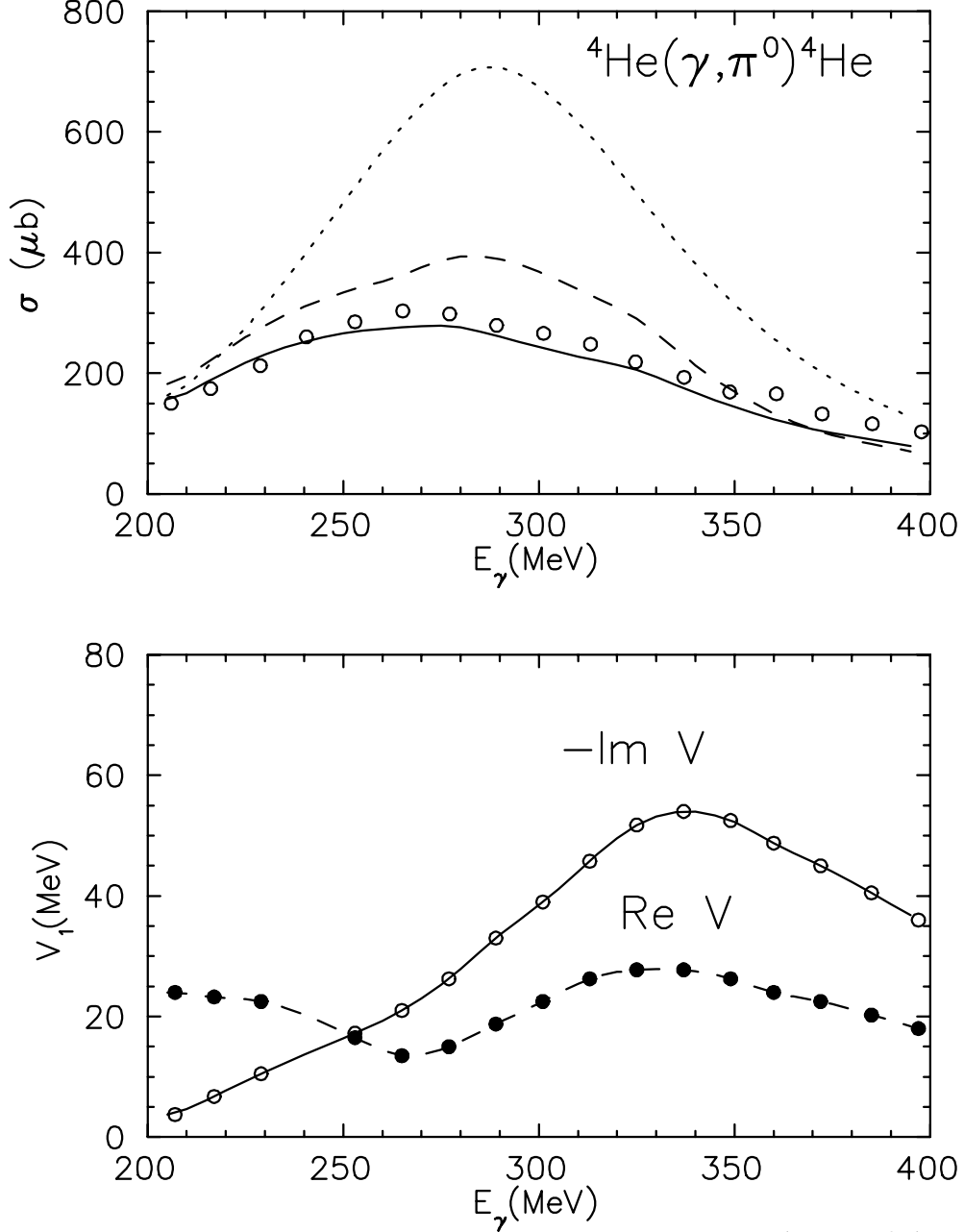


FIG. 5. Energy dependence of the total cross section for the ${}^4\text{He}(\gamma, \pi^0){}^4\text{He}$ reaction (upper figure) and the corresponding values of the F -type Δ -self energy $V_1(E_\gamma)$ from Eq. (26a) (lower figure). In the upper figure, dotted and dashed curves are the total cross sections for PWIA and DWIA, respectively. The solid curve is the result obtained with F -type Δ self-energy. Experimental data are from Ref. [30].

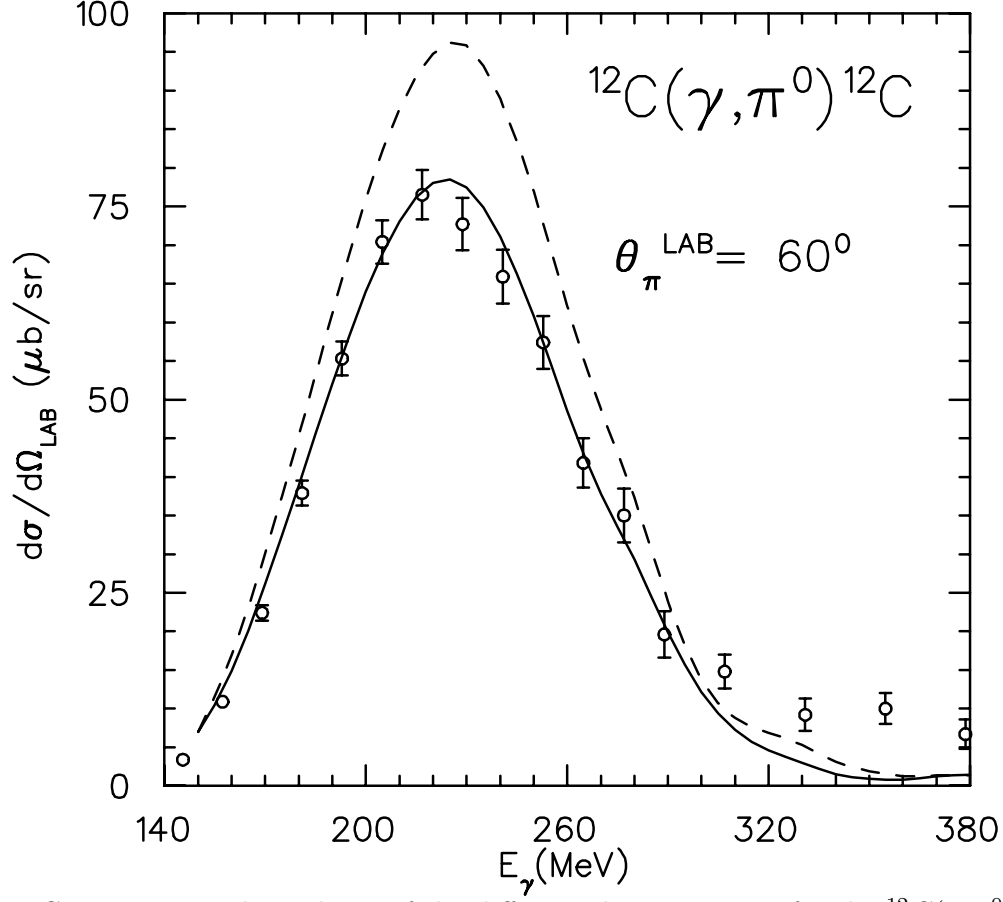


FIG. 6. Energy dependence of the differential cross section for the $^{12}\text{C}(\gamma, \pi^0)^{12}\text{C}$ reaction at pion *lab* angle $\theta_\pi = 60^\circ$. Notation for the curves as in Fig. 3. Experimental data are from Ref. [37].

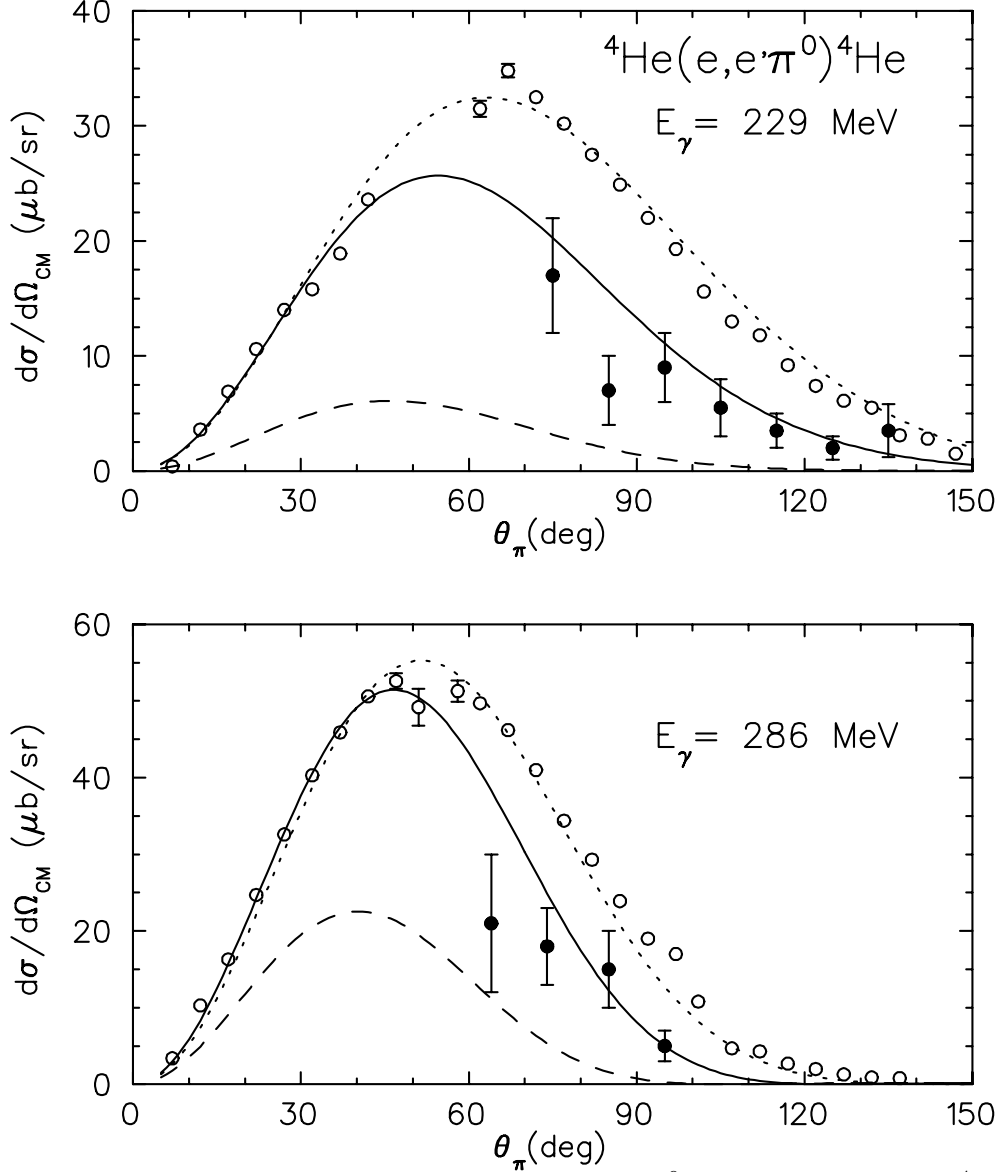


FIG. 7. Differential cross section for the coherent π^0 electroproduction on ^4He calculated with F -type Δ self energy, Eq. (26a). The dotted curves are the results at the photon point ($Q^2 = 0$). The corresponding experimental data are from Ref. [30] (open circles). The solid, and dashed curves in the upper (lower) figure are the results for $Q^2=0.062$ (0.054) and 1.81 (1.60) $(\text{GeV}/c)^2$, respectively. The experimental data [38] (full circles) correspond to $Q^2 = 0.062(\text{GeV}/c)^2$ in the upper figure and $Q^2 = 0.054(\text{GeV}/c)^2$ in the lower figure.